MODEL THEORY OF MODULES, ALGEBRAS AND CATEGORIES

Ettore Majorana Foundation and Centre for Scientific Culture

Erice (Trapani), Sicily, ITALY, July 28 – August 2, 2017

BOOK OF ABSTRACTS AND CONFERENCE PROGRAMME

Directors: Sonia L'Innocente, Alberto Facchini and Marcus Tressl

XII Course of the School of Statistical Physics.
Directors: Peter Hanggi, Fabio Marchesoni

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On the first page: view from Venere Castle, Monte Cofano, Erice

Welcome

We address a very warm welcome to all the participants and contributors to the MOTMAC Conference on “Model Theory of Modules, Algebras and Categories”.

The Conference is held in the superb location of Erice, hosted by the Ettore Majorana Foundation and Centre for Scientific Culture.

The main purpose of this conference is to offer an overview of the latest scientific advances in module theory with emphasis on the use of geometrical, topological, combinatorial and model theoretical methods. The subjects include model theory and module theory, representation theory of algebras, homological algebra, category theory.

The conference is dedicated to Prof. Mike Prest from the University of Manchester, on the occasion of his 65th birthday. His research has significant impact in these subjects.
MOTMAC will provide the opportunity for an integrated approach to solve problems related to decidability issues, domestic string algebras, tilting modules.

We hope that the conference will generate new ideas, contacts and collaborations within and accross the addressed subjects.

Alberto Facchini, Sonia L’Innocente, Marcus Tressl

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CONFERENCE PROGRAMME

	Fri Jul 28	Sat Jul 29	Sun Jul 30	Mon Jul 31	Tue Aug 1	Wed Aug 2
08.45-09.00	Arrival	Welcome				Departure
09.00-9.50		**Macintyre**	**Toffalori**	**Angeleri-Hügel**	**Trlifaj**	
10.00-10.50		**Herzog**	**Rothmaler**	**Otero** / **D'Este**	**Guil Asensio**	
11.00-11.30		Coffee Break				
11.30-12.20		**Salce**	**Point**	**Albu**	**Ziegler**	
12.30-13.00			Memory of **Puninski**			
		Lunch				
15.30-16.20		**Facchini**	Excursion	**Gregory**	**Bazzoni**	
16.30-16.50		**Nazemian**		**Kucera**	**Laking**	
17.00-17.30		Coffee Break		Coffee Break		
17.30-17.50	Informal meeting at San Rocco's Cloister	**Mantese**		**Tarantino**	**Vitória**	
18.00-18.20		**Saroch**		**Garkusha (18.00-18.50)**	**Marks**	
18.30-19.20		**Prihoda**			**Prest**	
20.30	Dinner	Dinner	Dinner	Social Dinner	Dinner	

Plenary talks: 50 minutes plus 5 minutes for questions.

Contributed talks: 20 minutes plus 5 minutes for questions.

FRIDAY 28

21.00
Happy Birthday, Mike!

SATURDAY 29

9.00-9.50
Angus Macintyre, *TBA*
10.00-10.50
Ivo Herzog *The universal quantum logic of a ring with involution*
11.30-12.20
Luigi Salce, *The addition theorem for algebraic entropies of endomorphisms of modules*

15.30-16.20
Alberto Facchini, *Modules with chain conditions up to isomorphism*
16.30-16.50
Zahra Nazemian, *Isoradical of modules and modules geenrated by isosimple modules*
*1*7.30-17.50
Francesca Mantese, *Prüfer modules over Leavitt path algebras*
18.00-18.20
Jan Saroch, $\sum$*-cotorsion modules, definability and singular compactness*
18.30-19.20
Pavel Prihoda, *On the Gorenstein defect categories*

SUNDAY 30

9.00-9.50
Carlo Toffalori, Decidability and modules over Bézout domains
10.00-10.50
Philipp Rothmaler, Strongly pure submodules of product of strict Mittag-Leffler mod**ules**

11.30-12.20

Françoise Point, Bézout domains and lattice-valued modules.

12.30-13.30
Memory of **Gennadi Puninski**

MONDAY 31

9.00-9.50
Livia Angeleri Hügel, *Torsion pairs in silting theory*
10.00-10.20
M. Otero, *Lie correspondence in definable groups* / **G. D'Este**, *Auslander-Reiten sequences and intuition*
11.30-12.20
T. Albu, *The Osofsky-Smith Theorem in rings, modules, categories, torsion theories, and lattices*

.30-16.20
Lorna Gregory, *Tubular algebras have decidable theory of modules*
16.30-16.50
Thomas Kucera, *The elementary socle series: problems and questions*
17.30-17.50
Marco Tarantino, *Recollements from cotorsion pairs*
18.00-18.50
Grigory Garkusha, *Derived categories for Grothendieck categories of enriched functors*

TUESDAY 1

9.00-9.50
Jan Trlifaj, *Locality for quasi-coherent sheaves associated with tilting*
10.00-10.20
Pedro Guil Asensio, *TBA*
11.30-12.20
Martin Ziegler, *Model Theory of right-angled buildings*

15.30-16.20
Silvana Bazzoni, *Periodic modules and acyclic complexes*
16.30-16.50
Rosanna Laking, *Locally coherent hearts and elementary cogenerators*
17.30-17.50
Jorge Vitória, *Cosilting classes in derived categories*
18.00-18.20
Frederik Marks, *Finite-type results in silting theory*
18.30-19.20
Mike Prest, *All sorts of modules*

Trapani

PLENARY TALKS

The Osofsky-Smith Theorem in rings, modules, categories, torsion theories, and lattices

Toma Albu

Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

The renown Osofsky-Smith Theorem (O-ST), invented in 1991, says that a cyclic (finitely generated) right R-module such that all of its cyclic (finitely generated) sub- factors are CS modules is a finite direct sum of uniform submodules.

In this talk we present various extensions of this theorem to Grothendieck categories (the Categorical O-ST), module categories equipped with a hereditary torsion theory (the Relative O-ST), and modular lattices (the Latticial O-ST); it illustrates a general strategy which consists on putting a module-theoretical concept/result into a latticial frame (we call it latticization) in order to translate that concept/result to Grothendieck categories (we call it absolutization) and module categories equipped with a hereditary torsion theorie (we call it relativization).

Torsion pairs in silting theory

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In a compactly generated triangulated category, we consider t-structures induced by silting objects or by their duals, cosilting objects. We show that the heart of such t-structures is a Grothendieck category if and only if the (co)silting object satisfies a purity assumption. In the cosilting case, the previous conditions are also related to the coaisle of the t-structure being a definable subcategory. As an application, the heart of any nondegenerate compactly generated t-structure is a Grothendieck category.

References:

1) Angeleri Hügel, L.; Marks, F.; Vitória, J. Torsion pairs in silting theory, Pacific Journal of Mathematics}, to appear; also arxiv:1611.08139.

TBA

Pedro Guil Asensio

Periodic modules and acyclic complexes

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We study the behavior of modules M that fit into a short exact sequence $0 \to M \to C \to M \to 0$, where C belongs to a class C of modules, the so-called C-periodic modules.

We find a rather general framework to improve and generalize some well-known results of Benson and Goodearl and Simson. In the second part we will combine techniques of hereditary cotorsion pairs and presentation of direct limits, to conclude, among other applications, that if M is any module and C is cotorsion, then M will be also cotorsion.

This will lead to some meaningful consequences in the category Ch(R) of unbounded chain complexes and in Gorenstein homological algebra.

References:

1) Benson, D. J.; Goodearl, K. R., Periodic flat modules, and flat moules for finite groups, Pacific J. Math. 2000, 196, 45-67..
2) Simson, D., Pure-periodic modules and a structure of pure-projective resolutions, Pacific J. Math. 2002, 207, 235-256.

Modules with chain conditions up to isomorphism

A. Facchini[a], Z. Nazemian[b]

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The content of the first of the two papers in the references will be presented. The second paper will be presented by my coauthor. We have studied modules with chain conditions up to isomorphism, in the following sense. Let R be a ring and M be a right R-module. We say that M is *isoartinian* if, for every descending chain $M \geq M_1 \geq M_2 \geq \dots$ of submodules of M, there exists an index $n \geq 1$ such that M_n is isomorphic to M_i for every $i \geq n$. Dually, we say that M is *isonoetherian* if, for every ascending chain $M \leq M_1 \leq M_2 \leq \dots$ of submodules of M, there exists an index $n \geq 1$ such that M_n is isomorphic to M_i for every $i \geq n$. Similarly, we say that M is *isosimple* if it is non-zero and every non-zero submodule of M is isomorphic to M. We study these three classes of modules, determining a number of their properties. The results we obtain give a good description of these modules and often have a surprising analogy with the "classical" case of artinian, noetherian and simple modules. For instance, we prove that any isoartinian module contains an essential submodule that is a direct sum of isosimple modules. The endomorphism ring of an isosimple module M_R is a right Ore domain, whose principal right ideals form a noetherian modular lattice with respect to inclusion. We say that a ring R is *right isoartinian* if the right module R_R is isoartianian. Several results will be presented.

References:

1) Facchini, A.; Nazemian, Z. *J. Algebra* **2016**, *453*, pp 578-pp 601.
2) Facchini, A.; Nazemian, Z. *J. Algebra* **2017**, *484*, pp 66-pp 87.

Derived categories for Grothendieck categories of enriched functors

Grigory Garkusha, Darren Jones

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Grothendieck categories of enriched functors occur in various fields of Algebra, Algebraic Geometry and Voevodsky's motivic homotopy theory. We investigate the derived category for a Grothendieck category of enriched functors and give explicit conditions when it is compactly generated triangulated. As an application, we construct many examples of compactly generated triangulated categories which are of independent interest.

Tubular algebras have decidable theory of modules

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A long standing conjecture of Mike Prest claims that a finite-dimensional algebra has decidable theory of modules if and only if it is of tame representation type. Up until recently, all known examples of finite-dimensional algebras with decidable theory of modules were of tame domestic representation type; whereas tubular algebras are tame but not domestic representation type.

A finite-dimensional k-algebra R is of tame representation type roughly if for each dimension d, all but finitely many indecomposable R-modules of dimension d are contained in finitely many 1-parameter families over k. A k-algebra is of domestic representation type if there is a bound on the number of 1-parameter families needed as d varies.

We say that a ring R has decidable theory of module if there is an algorithm which, when given a first-order statement about R-modules, answers, after a finite amount of time, whether this statement is true in all R-modules or not.

In this talk, I will explain in detail what it means for a theory of modules to be decidable and explain what domestic, tame and wild representation type are. I will then discuss a proof that tubular algebras have decidable theory of modules.

References:

Lorna Gregory, Decidability of theories of modules over canonical algebras of tubular type, arXiv:1603.03284

The universal quantum logic of a ring with involutio

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We will give an introduction to and discuss several aspects of the model theory of modules over a ring *(R,*)* with involution. Chief among these is the involution induced on the lattice of positive primitive formulae. There is a smallest congruence modulo which this involution induces an orthocomplemented modular quotient lattice, the universal quantum logic of *(R,*)*.

The ring of definable scalars is a *-regular ring and we will explain how it may be obtained, alternatively, using a technique introduced by Olivier in the 60's to construct the commutative regularization if a commutative ring.

TBA

A. Macintyre

Bézout domains and lattice-valued modules.

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We introduce a notion of ℓ-valued modules over a commutative Bézout domain **B**, an instance being the ring **B** endowed with the ℓ-valuation map to its lattice-ordered group of divisibility. This valuation map endows any **B**-module with a lattice of subgroups.
We obtain in particular a Feferman-Vaught type theorem for the class $C^{\mathbf{B}}_{\mathrm{tf}}$ of torsion-free **B**-modules. Let MSpec(**B**) be the maximal spectrum of **B**. We analyse the definable sets in terms, on one hand, of the definable sets in the classes $C_{\mathrm{tf}}{}^{\mathbf{B}_M}$, where $\mathbf{B}_M$ ranges over the localizations of **B** at M, where M belongs to MSpec(**B**), and on the other hand, of the constructible subsets of MSpec(**B**). When **B** has good factorization, it allows us to derive decidability results for the class $C_{tf}^{\mathbf{B}}$, in particular when **B** is the ring $\mathbf{Z}^{\sim}$ of algebraic integers or the one of rings $\mathbf{Z}^{\sim} \cap \mathbf{IR}$, $\mathbf{Z}^{\sim} \cap \mathbf{Q}_p$.
If time permits, we will also examine the case of the ring of holomorphic functions over **C** or the integral closure of that ring.
This is a joint work with Sonia L'Innocente.

All sorts of modules

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It has turned out that the mathematics in, and flowing from, the model theory of modules applies to a great variety of additive structures, usually requiring an expansion of the notion of module to allow for many sorts. In addition, it is useful to expand even normal, 1-sorted, modules to include further pp-defined sorts. I will illustrate this with various examples and applications.

Pure projective modules over serial rings

P. Příhoda[a]

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R.B. Warfield[3)] proved that every finitely presented module over a serial ring is a direct sum of uniserial modules. He asked whether the Krull-Remak-Schmidt theorem holds in the category of finitely presented modules over a serial ring. This question was answered in the negative by A. Facchini[1)].

In the talk I will consider similar questions for pure projective modules over a serial ring: Over which serial rings is every pure projective module a direct sum of finitely presented modules? When does the Krull-Remak-Schmidt-Azumaya theorem hold for pure projective modules over a serial ring? So far these questions were answered for chain domains – the Krull-Remak-Schmidt-Azumaya theorem for pure projectives over a chain domain holds if and only if there are no non-trivial idempotent ideals generated by 1 element[2)]. It appears that some ideas from the proof of this result can be transferred to the case of pure projective modules over serial rings.

References:

1) Facchini, A. Krull-Schmidt fails for serial modules, *Trans. Amer. Math. Soc.,* **1996**, *348*, 4561-4576.

2) Příhoda, P.; Puninski, G. Pure projective modules over chain domains with Krull dimension, *J.Algebra* **2016**, *459*, 189-212.

3) Warfield, R.B. Serial rings and finitely presented modules, *J. Algebra,* **1975**, *37*, 187-222.

Strongly pure submodules of product of strict Mittag-Leffler modules

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Abstract:

The addition theorem for algebraic entropies of endomorphisms of modules

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The Addition Theorem for algebraic entropies induced by length functions L on the category of locally L-finite modules over arbitrary rings is presented and its relevance for the whole theory of algebraic entropies is illustrated. A sketch of the proof and examples of non-discrete length functions over valuation domains and their induced algebraic entropies are provided (joint work with Simone Virili).

Decidability and modules over Bézout domains

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The model theory of modules over Bézout domains, with some hints at Prüfer domains, was developed in [PT2]. These results were useful in showing the decidability of the first order theory of modules over the ring of algebraic integers [LPT] and over Bézout domains obtained from principal ideal domains via the so called D+M-construction [PT1].

On the other hand Lorna Gregory [G], extending [PPT], proved that the theory of modules over a(n effectively given) valuation domain V is decidable if and only if there is an algorithm that decides the prime radical relation in V, namely, for every a, b in V, answer whether a is in the radical of bV, equivalently whether the prime ideals of V containing b also include a.

In this talk we propose a similar analysis over (effectively given) Bézout domains, and even Prüfer domains, with infinite residue fields with respect to maximal ideals. Recall that a domain is Prüfer if and only if all its localizations at maximal ideals are valuation domains. Bézout domains are a notable subclass. Bézout domains with infinite residue fields include the ring of algebraic integers and the ring of complex valued entire functions - actually the latter is uncountable, but an analysis of its Ziegler spectrum is given in [LPPT]. However other noteworthy, countable and effectively given examples can be provided.

Our main result says that, if B is an effectively given Bézout domain with infinite residue fields, then the theory of B-modules is decidable if and only if there is algorithm which answers a sort of double prime radical relation, in detail, given a, b, c, d in B, decides whether, for all prime ideals P and Q of B with P+Q different from R, b in P implies a in P or d in Q implies c in Q. Generalizations to Prüfer domains will be also discussed.

This research was developed with Lorna Gregory, Sonia L'Innocente and Gena Puninski [GLPT]. I'd like to dedicate this talk to the memory of Gena.

References

[G] L. Gregory, Decidability for theories of modules valuation domains, J. Symbolic Logic 80 (2015), 684-711

[GLPT] L. Gregory-S. L'Innocente-G. Puninski-C. Toffalori, Decidability of the theory of modules over Bézout domains with infinite residue fields, arXiv:1706.08940 [math.LO]

[LPT] S. L'Innocente-G. Puninski-C. Toffalori, On the decidability of the theory of modules over the ring of algebraic integers, Ann. Pure Applied Logic 168 (2017), 1507-1516

[LPPT] S. L'Innocente-F. Point-G. Puninski-C. Toffalori, The Ziegler spectrum of the ring of entire complex valued functions, arXiv:1703.01752 [math.LO]

[PPT] G. Puninski-V. Puninskaya-C. Toffalori, Decidability of the theory of modules over commutative valuation domains, Ann. Pure Applied Logic 145 (2007), 258-275

[PT1] G. Puninski-C. Toffalori, Decidability of the theory of modules over a Bézout domain D + X Q[X] with D a principal ideal domain and Q its field of fractions, J. Symbolic Logic 79 (2014), 296-305

[PT2] G. Puninski-C. Toffalori, Some model theory of modules over Bézout domains. The width, J. Pure Applied Algebra 219 (2015), 807-82

Locality for quasi-coherent sheaves associated with tilting

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By the classic work of Raynaud and Gruson [6], the notion of a vector bundle is Zariski local for any scheme. In the formalism of [3], one can view vector bundles as particular instances, for n = 0, of the more general notions of quasi-coherent sheaves induced by n-tilting modules. Using relations between tilting and Mittag-Leffler conditions from [1], and the recent classification of tilting classes over commutative rings [2], [4], we show that also these general notions are Zariski local, for all n and all schemes (joint work with Michal Hrbek and Jan S˘´tov´ı˘cek, [5]).

References

1)[1] L. Angeleri Hu¨gel, D. Herbera, Mittag-Leffler conditions on modules, Indiana Math. J. 57(2008), 2459-2517.

2)[2] L. Angeleri Hu¨gel, D. Posp´ı˘sil, J. S˘˘tov´ı˘cek, J. Trlifaj, Tilting, cotilting, and spectra of commutative noetherian rings, Trans. Amer. Math. Soc. 366(2014), 3487-3517.

3)[3] E.E. Enochs, S. Estrada, Relative homological algebra in the category of quasi-coherent sheaves, Adv. Math. 194(2005), 284-295.

4)[4] M. Hrbek, J. S˘˘tov´ı˘cek, Tilting classes over commutative rings, preprint, arXiv:1701.05534v1.

5)[5] M. Hrbek, J. S˘˘tov´ı˘cek, J. Trlifaj, Locality of quasi-coherent sheaves associated with tilting, preprint.

6)[6] M. Raynaud, L. Gruson, Crit`eres de platitude et de projectivit´e, Invent. Math. 13(1971), 1 - 89.

Model theory of right-angled buildings

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For a right-angled Coexter group G let B(G) the associated countable Tits building with infinite residues. Using a suitable language, we study the first order theory of B(G). It has a nice axiomatization, is omega-stable, equational and has trivial forking.

Salt pan Mozia, Lagoon of Marsala.

CONTRIBUTED TALKS

On the Gorenstein Defect Categories

Javad Asadollahi[a]

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In this talk, we study several triangulated categories which detect Gorensteinness in the sense that their triviality implies the Gorenstein property. These triangulated categories have a long history, starting from the famous unpublished paper by Buchweitz from the mid-eighties, and the last decade became popular in Commutative Algebra, Representation Theory, and Algebraic Geometry, through the works of [3], [2], [1] and others, via various relative triangulated categories of singularities.

One of the main themes is the study of the relationship between the homotopy category of acyclic complexes of projectives and the homotopy category of totally acyclic complexes of projectives. In various interesting cases, the coincidence of these homotopy categories characterise the Gorenstein property. We show that a virtually Gorenstein artin algebra is Gorenstein if and only if any acyclic complex of projectives is totally acyclic.

The talk is based on a joint work with R. Hafezi and T. Dehghanpour.

References:

1) Bergh, P. A.; Jorgensen, D. A.; Oppermann, S. The Gorenstein defect category, Q J Math (**2015**) 66 (2): 459-471.
2) Iyengar, S; Krause, H. Acyclicity versus total acyclicity for complexes over Noetherian rings, Documenta Math, 11 (**2006**), 207-240.
3) Orlov, D. Triangulated categories of singularities and D-branes in Landau-Ginzburg models. Proc. Steklov Inst. Math. (**2004**), 246(3), 227–248.

Auslander-Reiten sequences and intuition

Gabriella D'Este

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Auslander-Reiten sequences (or almost split sequences) are short exact sequences which do not split, but very close to split exact sequences.

P. Gabriel pointed out the following at the end of [1] : "The name is a dedication of Ringel to the authors of almost split sequences. He introduced Auslander-Reiten quivers in his Brandeis lectures (1975) and determined their structure for tame and wild quivers. Since then various specialists like Bautista, Brenner, Butler, Riedtmann have hoarded a few hundred examples in their dossiers, thus getting an intuition which no theoretical argument can replace."

In this talk I will present more or less complicated Auslander-Reiten sequences of modules, defined over finite dimensional algebras of finite or infinite representation type. I will also compare Auslander-Reiten sequences with the same first or last non zero term, computed in different module categories.

As we shall see, with respect to suitable bases, the socalled irreducible maps are a kind of "additions" or "cancellations".

References:

[1] P. Gabriel, Auslander - Reiten sequences and representation-finite algebras, Springer LNM 831 (1980) 1 - 71 .

The elementary socle series: problems and questions

T. G. Kucera[a]

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I discuss the background for work planned for a sabbatical leave visit with Mike Prest in Manchester, problems that I have been struggling with for almost 20 years, cf. Kucera[3].

The *elementary socle series* of a module was introduced by Ivo Herzog[1] as a natural definable analog of the algebraic socles (*Loewy series*). If *N* is a totally transcendental (tt) module, then the elementary socle series is well defined and increasing, and for some ordinal α, $soc^{\alpha}(N) = N$, that is, the elementary socle series *exhausts N*. An important family of examples of tt modules are the injective (right) modules over a (right) noetherian ring. In ring theory, these can be studied by the the *Fundamental Series* of A. V. Jategaonkar[2]. Neither the socle series nor the fundamental series necessarily exhausts the module under consideration.

Each term of the elementary socle series of a tt module *N* is a definably closed, fully invariant submodule of *N*. [It was certainly known to Herzog that the elementary socles are submodules; Prest and I showed the more general result years ago, cf. Kucera[3].] This is virtually all that is known about the elementary socles in general.

The classic example of a structure theory is that given by Matlis[4] for indecomposable injective modules over a commutative noetherian ring; there is also a well-developed structure theory for certain non-commutative noetherian rings based on the fundamental series.

There is a one-to-one correspondence between direct sum indecomposable tt modules *N* and non-orthogonality classes C of strongly regular one-types. A good structure theory should describe the structure of *N* in terms of properties of *C* (in some sort of analogy to the Matlis or Jategaonkar results). The prime question of interest would be a description of the factors $soc^{\alpha+1}(N)/soc^{\alpha}(N)$ for all α; but also interesting are the factors $N/soc^{\alpha}(N)$. I do not know an example where the latter is not a tt module.

References:

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Locally coherent hearts and elementary cogenerators

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By results of Angeleri-Hügel, Marks and Vitória[1], we know that a t-structure in a compactly generated triangulated category induced by a pure-injective cosilting object **C** has a Grothendieck heart $\mathbf{G}_{\mathbf{C}}$.

We consider the question of when the heart satisfies additional finiteness conditions. Using that $\mathbf{G}_{\mathbf{C}}$ may be realised as a localisation of a functor category, we first show that $\mathbf{G}_{\mathbf{C}}$ is locally noetherian if and only if **C** is sigma-pure-injective; this generalises the result for cotilting modules[2] to cosilting objects. Moreover, we show that $\mathbf{G}_{\mathbf{C}}$ is locally coherent if and only if **C** is an elementary cogenerator. Finally, we will consider the case where **C** is the image of an **n**-cotiliting module in the derived category of a module category. We will show that $\mathbf{G}_{\mathbf{C}}$ satisfies the above finiteness conditions exactly when **C** satisfies the corresponding purity conditions in the module category.

References:

1) Angeleri-Hügel, L.; Marks, F.; Vitória, J. 'Torsion pairs in silting theory' *to appear in Pacific J. Math*.

2) Colpi, R.; Mantese, F.; Tonolo, A. 'Cotorsion pairs, torsion pairs and Σ-pure-injective cotilting modules' *J. Pure Appl. Algebr.* **2010***, 214,* 519-525.

Prüfer modules over Leavitt path algebras

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Let $L_K(E)$ denote the Leavitt path algebra associated to the finite graph E and field K. For each closed path c in E we construct the uniserial, artinian, non-noetherian Prüfer module $U_{E,c}$, recalling the classical construction of Prüfer abelian groups. In our main result we give necessary and sufficient conditions on the graph E and on the path c for the injectivity of $U_{E,c}$. In this case, $U_{E,c}$ is precisely the injective hull of its unique simple composition factor. The result relies on a set of highly nontrivial tools, including: some general results about uniserial modules over arbitrary associative unital rings; a Division Algorithm in $L_K(E)$ with respect to the element $c - 1$; two types of Morita equivalences for Leavitt path algebras; and the fact that every Leavitt path algebra is Bezout (i.e., that every finitely generated one-sided ideal is principal).

This is a joint work with Gene Abrams, University of Colorado, and Alberto Tonolo, University of Padova.

Finite-type results in silting theory

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Silting complexes can be understood as derived analogues of tilting modules. They give rise to both a t-structure and a co-t-structure in the derived category which are adjacent, that is, the aisle of the t-structure turns out to be the coaisle of the co-t-structure. In this talk, we show that a co-t-structure in the derived category arises from a silting complex if and only if it is compactly generated and admits an intermediate coaisle.

Silting modules can be defined as cohomology groups of 2-term silting complexes. They give rise to torsion classes in the module category that are particularly relevant from an approximation theoretical point of view. We show that such torsion classes can be characterised by a finite-type condition. If time permits, we will also discuss applications of this result to localisations of rings.

This talk is based on joint work with Lidia Angeleri Hügel, Jan Stovicek and Jorge Vitória.

Isoradical of modules and modules generated by isosimple modules

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A module M is called isosimple if it is non-zero and every non-zero submodule of M is isomorphic to M. For a ring R, the right isoradical I-rad (R) of a ring R is defined to be the intersection of the annihilators of all isosimple right R-modules.

Unlike Jacobson radical, which is the intersection of the annihilators of simple right modules, isoradical of a ring is not left/right symmetric.

We generalize the concept of isoradical from rings to modules and using that we study modules generated by isosimple modules.

Special cases of such modules are when a module is a sum or a direct sum of isosimple modules. A module that is a sum of isosimple modules is not necessarily a direct sum of isosimple modules, but if a module M is a sum of pairwise non-isomorphic isosimple modules, then the sum is direct. A ring that is generated by isosimple right modules must be a semiprime right noetherian ring. It is shown that a ring R is generated by isosimple right modules if and only if it is a sum of isosimple right ideals, if and only if R is a finite direct product of prime right noetherian rings that are sums of isosimple right ideals. We do not know whether such a ring is a direct sum of isosimple right ideals.

References:

7) Facchini, A.; Nazemian, Z. Modules with chain conditions up to isomorphism, *J. Algebra 453,* 2016, 453, 578--601
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Lie correspondence in definable groups

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Let R be a o-minimal expansion of a real closed field. Let G be a group definable in R, and Lie(G), its Lie algebra. In general, due to the lack of an exponential map from Lie(G) to G, we cannot ensure the existence of a definable subgroup of G whose Lie algebra is a given subalgebra of Lie(G). We will give some examples of subalgebras for which such definable subgroups exist, most notably Cartan subalgebras of Lie(G).

References:

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Non reduced schemes and Zariski Geometries

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I will propose a model theoretic structure which aims to capture the algebra (or geometry) of a non reduced scheme over an algebraically closed field. This structure has quantifier elimination and its picture is similar to Quantum Zariski Geometries and other ones consider by M. Kamenski to prove model theoretic tameness of quasi coherent sheaves, the three of these approaches have a flavor of representation theory. I will talk about relations between definable sets in this structure and arbitrary closed sets in an non reduced scheme.

∑-cotorsion modules, definability and singular compactness

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We formulate a general statement which allows us to demonstrate, over a general ring R, that being a ∑-cotorsion R-module is a property of the complete theory of modules. In [1, Theorem 11], it was proved that, over a countable ring, a module C is ∑-cotorsion, if and only if every (countably presented) flat module is C-stationary. We show how to strengthen the C-stationary assumption in order to get the corresponding statement for modules over uncountable rings.

One of the tools used in proving our result is a new version of singular compactness theorem which does not use subobjects (in the general case).

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1) Guil Asensio, P. A.; Herzog, I. *Model-theoretic aspects of ∑-cotorsion modules, Ann. Pure Appl. Logic* **2007**, *146*, 1–12.

Recollements from Cotorsion Pairs

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It is possible to use complete cotorsion pairs in a Grothendieck category G to build model structures on the category of unbounded complexes $\mathrm{Ch}(G)$ and, in some circumstances, these model structures induce a localization of triangulated categories between the corresponding homotopy categories that actually fit in a recollement. In this talk, I will use the Hovey's bijection between abelian model structures and cotorsion pairs to explain this phenomenon and recover known examples of recollements as well as new ones.

Cosilting classes in derived categories

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In a module category, cotilting classes are known to be precisely the resolving and definable subcategories of the module category whose Ext-orthogonal class has bounded injective dimension. We discuss a derived analogue of this statement for cosilting complexes, namely the fact that cosilting classes are precisely the cosuspended, definable and co-intermediate subcategories of the derived category. Crucial for this statement is the fact that cosilting complexes are pure-injective. This is joint work with Frederik Marks.

Saint Caterina Castle, Cala Rossa Favignana Island (on of Egadi Islands).

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